

The Surprising “Alpha” from Malkiel’s Monkey and Upside-Down Strategies¹

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Abstract

Recent index literature is replete with innovations based on quantitative strategies that are predicated on sensible investment beliefs. Empirical studies confirm that these strategies deliver economically large and statistically significant excess returns over cap-weighted market benchmarks in nearly all regions and countries, over long periods of time. In this article, Arnott, Hsu, Kalesnik, and Tindall show that inverting these portfolio construction algorithms does not reverse the outperformance. Indeed, the “upside-down” strategies often outperform the originals. This paradoxical result is driven by the phenomenon that seemingly unrelated and non-*value*-based strategies and their inverted counterparts often have unintended and almost unavoidable value and small-cap tilts. Even Burt Malkiel’s legendary blind-folded monkey, throwing darts at the *Wall Street Journal*’s stock page, would produce a portfolio with a substantial value and small-cap bias that would have historically outperformed the S&P 500. The value and small-cap tilts stem from the fact that non-price-based weighting schemes sever the link between a company’s share price and its weight in the portfolio. Clearly, the inverted strategy of a non-price weighted strategy is still a non-price weighted strategy, would consequently have a value and small-cap tilt, and would therefore have outperformed historically.

Investment practitioners and many in academia are understandably preoccupied with identifying stock characteristics and strategies that can prospectively deliver high risk-adjusted returns. Many sensible investment beliefs, when translated into portfolio “weights,” result in historical outperformance relative to the cap-weighted benchmark. The naïve expectation is that, when we invert the weighting algorithms of these sensible investment heuristics, effectively turning them upside down, these inverted strategies should underperform by roughly as much as the original algorithm outperformed. Instead, we find that these upside-down strategies also beat the cap-weighted benchmark, often by more than the upright originals. Indeed, even a portfolio generated by Malkiel’s blindfolded monkey² throwing darts outperforms the market.³

How can it be that a monkey, who may have great skill with darts, but presumably has no skill in evaluating investments, adds value? Our findings suggest that the investment beliefs upon which many investment strategies are ostensibly based play little or no role in their outperformance⁴. This does not mean that the outperformance of these strategies is suspect. Rather, as it turns out, these investment beliefs work because they introduce, often unintentionally, value and small cap tilts into the portfolio. Counter-intuitively, when we invert these strategies, the resulting portfolios continue to display value and small cap bias. We demonstrate this paradoxical effect mathematically in the Appendix I.

The results we present are puzzling until one grasps the derivations in the literature starting with Berk [1997]. Berk [1997] and Arnott, Hsu, Liu, and Markowitz [2011] argue that size and value effects are created by low prices. Berk ascribes this to hidden risk factors; AHLM ascribes this to mean-reverting errors in price. Either way, falling prices lead to low book-to-market ratios and low market capitalization; whenever prices mean-revert, we observe value and small stocks outperform. Hsu [2006] and Arnott and Hsu [2008] offer a conceptual framework where non-price-weighted portfolios, which contra-trade against price changes at each rebalancing, necessarily result in value and size tilts, *regardless of the weighting method chosen*.

In summary, value and small cap exposures are naturally occurring portfolio characteristics, unless an investor constructs his portfolio to have positive relationship between price and portfolio weights. In this paper, we illustrate the above theoretical results with simple and easily replicable portfolio back-tests. We do not attempt to comment on the interesting debate regarding the nature of value and small cap premiums.

Research Design

This research is motivated by the proliferation of quasi-passive equity index strategies and their noteworthy long-term outperformance against traditional cap-weighted benchmarks in backtests, despite sometimes diametrically opposed investment beliefs. This leads to a natural skepticism; it's hard to believe that they could all work, in light of the long literature on the mean-variance efficiency of the cap-weighted benchmark and the underperformance of active management. However, the empirical evidence from domestic and global market data, extending back as far as data are available, suggests that the outperformance is robust.⁵

Our examination of this puzzle starts with portfolios formulated from an array of arguably sensible investment beliefs; we then invert them to create less intuitive strategies. In inverting the strategies, we tacitly examine whether these strategies outperform *because* they are predicated on meaningful investment theses and deep insights on capital markets, *or for reasons unrelated to the investment theses*. If the investment beliefs are the source of outperformance, then contradicting those beliefs should lead to underperformance.

For each of the investment beliefs, we create long-only equity portfolios using simple weighting heuristics; we then turn them “upside down”. For each of the quasi-index strategies, we form two inverse portfolios: (1) an Inverse Ratio portfolio, formed by normalizing the inverse weight $1/w$ and (2) an Inverse Complement portfolio, formed by normalizing the complementary weight $(\max(w)-w)$ of the original portfolio.⁶ Except for some special situations, the two inverse portfolios would generally have comparable characteristics. We also compute portfolios based on Malkiel's blindfolded dart-throwing monkey. We do not invert these portfolios, because the inverse of an equally-weighted portfolio is itself.

To ensure that we are investing in sufficiently liquid stocks, we restrict our universe to the largest 1,000 stocks by market capitalization in the United States.⁷ We extend the analysis to global markets both at an individual country level and a global portfolio level. For the global country portfolios, we use the largest stocks by market capitalization, matching the number of stocks to the most popular local cap-weighted benchmark indexes⁸; the global country results are generally similar qualitatively to the U.S. results, but often with a considerably larger magnitude of CAPM and Fama–French Four Factor model (FF4) alpha.⁹

All portfolios are rebalanced annually on the last trading day of the year. The portfolio schemes are back-tested using as much historical data as are available in the CRSP/CompuStat merged database for the United States, and the Worldscope and Datastream databases for other developed countries. When needed for portfolio construction, the risk parameters, such as variances and covariances, are estimated using the previous five years of monthly data. For example, for covariance-based strategy portfolio for 2003, we will use the sample covariance matrix from 1998–2002.¹⁰ Appendix II contains a summary of the strategies.

We break our analysis into five categories.

Reference portfolios

We establish three reference portfolios. Our first reference portfolio is the cap-weighted portfolio, which most people consider a reasonable representation of the market. **Exhibit 1** summarizes key attributes of the U.S. cap-weighted portfolio, using data from 1964–2012.

The second line of **Exhibit 1** displays the equal-weighted (EW) portfolio, which represents perhaps the strongest level of investor naiveté, tacitly believing that all stocks have identical expected returns and risk attributes. This makes EW an interesting and sensible secondary reference portfolio. One might also interpret EW as an effective approach for capturing stock price mean-reversion, where, at each rebalancing, the portfolio mechanically buys stocks that have fallen in price relative to others – unless they've fallen so far that they no longer make the size cut for the country – and sells stocks that have risen in price relative to others.

The EW portfolio produces 180 bps per year of incremental performance over the cap-weighted reference benchmark. This incremental performance is almost entirely due to a substantial Size and Value factor loading; EW delivers a 0.15% annualized FF4 alpha, with no statistical or economic significance. Throughout this paper, it will become increasingly clear why the EW portfolio is a sensible reference benchmark for other non-price-weighted strategy indices.

Exhibit 1. Performance Summary, Strategies, Inverse Strategies, and Random Portfolios, (United States, 1964–2012)

| | Strategy | Return | Standard Deviation | Sharpe Ratio | Value Added | Tracking Error | Information Ratio | CAPM Alpha | CAPM Beta | Alpha t-Stat | Annual FFC Alpha | Alpha t-stat | Market Exposure | Size Exposure | Value Exposure | Momentum Exposure |
|--|--|---------------|--------------------|--------------|--------------|----------------|-------------------|-------------|-------------|--------------|------------------|--------------|-----------------|---------------|----------------|-------------------|
| | US Cap Weighted | 9.66% | 15.29% | 0.29 | 0.00% | 0.00% | 0.00 | 0.00% | 1.00 | 0.00 | 0.00% | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 |
| | Equal Weight | 11.46% | 17.37% | 0.36 | 1.80% | 5.00% | 0.36 | 1.63% | 1.09 | 2.21 | 0.15% | 0.38 | 1.05 | 0.38 | 0.12 | -0.02 |
| High Risk = High Reward | Volatility Weighted | 12.15% | 19.13% | 0.36 | 2.49% | 7.24% | 0.34 | 1.98% | 1.17 | 1.91 | 0.23% | 0.46 | 1.10 | 0.55 | 0.16 | -0.04 |
| | Market Beta Weighted | 11.89% | 19.76% | 0.34 | 2.23% | 7.60% | 0.29 | 1.55% | 1.21 | 1.47 | 0.56% | 1.01 | 1.13 | 0.54 | 0.13 | -0.09 |
| | Downside Semi-Deviation Weighted | 12.13% | 18.92% | 0.37 | 2.47% | 6.90% | 0.36 | 1.99% | 1.16 | 2.02 | 0.26% | 0.52 | 1.10 | 0.52 | 0.17 | -0.04 |
| | Inverse-Ratio of Volatility Weighted | 12.53% | 15.64% | 0.47 | 2.86% | 5.36% | 0.53 | 3.24% | 0.96 | 3.97 | 0.58% | 1.13 | 0.97 | 0.28 | 0.33 | -0.03 |
| | Inverse-Complement of Volatility Weighted | 12.59% | 16.40% | 0.45 | 2.92% | 5.30% | 0.55 | 3.08% | 1.02 | 3.79 | 0.64% | 1.37 | 1.01 | 0.35 | 0.29 | -0.03 |
| | Inverse-Ratio of Market Beta Weighted | 13.48% | 15.02% | 0.55 | 3.81% | 7.22% | 0.53 | 4.58% | 0.87 | 4.30 | 0.86% | 1.07 | 0.91 | 0.25 | 0.43 | 0.03 |
| | Inverse-Complement of Market Beta Weighted | 12.63% | 16.16% | 0.46 | 2.97% | 5.35% | 0.55 | 3.20% | 1.00 | 3.89 | 0.48% | 1.01 | 0.99 | 0.34 | 0.31 | -0.01 |
| | Inverse-Ratio of Downside Semi-Deviation Weighted | 12.45% | 15.62% | 0.46 | 2.78% | 5.30% | 0.53 | 3.16% | 0.96 | 3.91 | 0.48% | 0.95 | 0.97 | 0.28 | 0.33 | -0.02 |
| | Inverse-Complement of Downside Semi-Deviation | 12.51% | 16.04% | 0.45 | 2.84% | 5.25% | 0.54 | 3.09% | 0.99 | 3.85 | 0.51% | 1.04 | 0.99 | 0.31 | 0.31 | -0.02 |
| Optimization-Based | Minimum Variance | 11.75% | 11.69% | 0.56 | 2.09% | 8.04% | 0.26 | 3.77% | 0.65 | 4.06 | 1.05% | 1.39 | 0.70 | 0.13 | 0.34 | 0.00 |
| | Maximum Diversification | 11.99% | 13.96% | 0.48 | 2.32% | 6.58% | 0.35 | 3.28% | 0.82 | 3.57 | 0.40% | 0.54 | 0.83 | 0.26 | 0.26 | 0.04 |
| | Risk-Efficient ($\lambda=2$) | 12.50% | 16.81% | 0.43 | 2.83% | 5.35% | 0.53 | 2.87% | 1.04 | 3.52 | 0.63% | 1.32 | 1.03 | 0.36 | 0.26 | -0.03 |
| | Risk Cluster Equal Weight | 11.18% | 14.61% | 0.41 | 1.51% | 4.92% | 0.31 | 2.13% | 0.91 | 2.95 | 0.31% | 0.49 | 0.94 | 0.03 | 0.21 | 0.03 |
| | Inverse-Ratio of Minimum Variance | 12.66% | 18.14% | 0.41 | 2.99% | 6.29% | 0.48 | 2.70% | 1.12 | 2.93 | 0.54% | 1.07 | 1.08 | 0.45 | 0.25 | -0.04 |
| | Inverse-Complement of Minimum Variance | 12.51% | 17.41% | 0.42 | 2.85% | 5.83% | 0.49 | 2.74% | 1.08 | 3.13 | 0.47% | 0.98 | 1.05 | 0.41 | 0.26 | -0.04 |
| | Inverse-Ratio of Maximum Diversification | 12.48% | 17.58% | 0.41 | 2.82% | 6.01% | 0.47 | 2.68% | 1.08 | 2.97 | 0.52% | 0.94 | 1.07 | 0.38 | 0.28 | -0.05 |
| | Inverse-Complement of Maximum Diversification | 12.37% | 17.30% | 0.41 | 2.71% | 5.70% | 0.48 | 2.63% | 1.07 | 3.06 | 0.36% | 0.76 | 1.05 | 0.40 | 0.26 | -0.03 |
| | Inverse-Ratio of Risk-Efficient ($\lambda=2$) | 12.35% | 17.32% | 0.41 | 2.68% | 5.81% | 0.46 | 2.61% | 1.07 | 2.97 | 0.25% | 0.51 | 1.04 | 0.41 | 0.27 | -0.03 |
| | Inverse-Complement of Risk-Efficient ($\lambda=2$) | 12.34% | 17.53% | 0.41 | 2.67% | 5.96% | 0.45 | 2.55% | 1.08 | 2.85 | 0.21% | 0.41 | 1.05 | 0.42 | 0.26 | -0.03 |
| | Inverse-Ratio of RCEW | 13.23% | 18.96% | 0.42 | 3.57% | 8.98% | 0.40 | 3.37% | 1.10 | 2.48 | -0.16% | -0.19 | 1.06 | 0.62 | 0.41 | -0.02 |
| | Inverse-Complement of RCEW | 12.43% | 17.21% | 0.42 | 2.76% | 5.68% | 0.49 | 2.71% | 1.06 | 3.15 | 0.41% | 0.85 | 1.04 | 0.40 | 0.26 | -0.03 |
| Fundamentals-Based | Book Value Weighted | 11.23% | 15.66% | 0.38 | 1.57% | 4.51% | 0.35 | 1.87% | 0.98 | 2.71 | 0.54% | 1.56 | 1.03 | 0.03 | 0.34 | -0.10 |
| | 5yr avg Earnings Weighted | 11.18% | 15.08% | 0.40 | 1.52% | 4.16% | 0.36 | 1.95% | 0.95 | 3.11 | 0.64% | 1.92 | 1.00 | 0.00 | 0.31 | -0.08 |
| | Fundamental Weighted | 11.60% | 15.45% | 0.41 | 1.93% | 4.64% | 0.42 | 2.30% | 0.96 | 3.26 | 0.64% | 1.83 | 1.01 | 0.05 | 0.37 | -0.09 |
| | Earnings Growth Weighted | 12.42% | 19.03% | 0.38 | 2.76% | 7.26% | 0.38 | 2.29% | 1.16 | 2.19 | 0.96% | 1.34 | 1.09 | 0.47 | 0.04 | 0.00 |
| | Inverse-Ratio of Book Value Weighted | 13.86% | 18.52% | 0.47 | 4.19% | 8.22% | 0.51 | 4.03% | 1.09 | 3.24 | 1.39% | 2.14 | 1.05 | 0.56 | 0.39 | -0.11 |
| | Inverse-Complement of Book Value Weighted | 13.04% | 17.49% | 0.45 | 3.38% | 6.55% | 0.52 | 3.33% | 1.06 | 3.35 | 1.09% | 2.05 | 1.05 | 0.39 | 0.37 | -0.11 |
| | Inverse-Ratio of 5yr avg Earnings Weighted | 14.38% | 18.34% | 0.50 | 4.71% | 8.58% | 0.55 | 4.66% | 1.06 | 3.56 | 1.65% | 2.19 | 1.03 | 0.57 | 0.41 | -0.09 |
| | Inverse-Complement of 5yr avg Earnings Weighted | 13.16% | 17.08% | 0.47 | 3.50% | 6.44% | 0.54 | 3.56% | 1.04 | 3.62 | 1.12% | 2.00 | 1.03 | 0.37 | 0.38 | -0.09 |
| | Inverse-Ratio of Fundamental Weighted | 14.06% | 18.77% | 0.47 | 4.39% | 8.63% | 0.51 | 4.21% | 1.10 | 3.22 | 1.40% | 2.06 | 1.05 | 0.60 | 0.41 | -0.11 |
| | Inverse-Complement of Fundamental Weighted | 13.34% | 17.60% | 0.46 | 3.67% | 6.89% | 0.53 | 3.63% | 1.06 | 3.47 | 1.19% | 2.11 | 1.05 | 0.41 | 0.40 | -0.11 |
| | Inverse-Ratio of Earnings Growth Weighted | 10.26% | 18.05% | 0.28 | 0.59% | 5.64% | 0.10 | 0.26% | 1.13 | 0.33 | -0.95% | -2.17 | 1.07 | 0.42 | 0.10 | -0.02 |
| | Inverse-Complement of Earnings Growth Weighted | 11.37% | 17.27% | 0.36 | 1.70% | 4.90% | 0.35 | 1.55% | 1.09 | 2.14 | 0.08% | 0.20 | 1.04 | 0.37 | 0.13 | -0.02 |
| Average of 100 Malkiel's Monkey Portfolios | 11.26% | 18.34% | 0.33 | 1.60% | 7.76% | 0.21 | 1.43% | 1.09 | 1.22 | -0.29% | -0.31 | 1.05 | 0.37 | 0.13 | -0.02 | |
| Average for Non-Cap-Weight Strategies, excl. Inverses | 11.75% | 16.60% | 0.40 | 2.09% | 6.15% | 0.35 | 2.23% | 1.02 | 2.63 | 0.47% | 0.96 | 1.00 | 0.28 | 0.22 | -0.03 | |
| Average for All Inverse-Ratio Strategies | 12.88% | 17.45% | 0.44 | 3.22% | 6.91% | 0.46 | 3.23% | 1.05 | 3.08 | 0.60% | 0.88 | 1.03 | 0.44 | 0.33 | -0.05 | |
| Average for All Inverse-Complement Strategies | 12.57% | 17.04% | 0.43 | 2.91% | 5.80% | 0.50 | 2.91% | 1.05 | 3.30 | 0.60% | 1.16 | 1.03 | 0.38 | 0.29 | -0.05 | |

Source: Research Affiliates based on CRSP/Compustat data.

Favoring high risk stocks in our portfolios

Given the theoretical and empirical links between risk and return, one might expect higher returns to be linked to stocks with higher risk. A naïve way to act on this belief, for investors willing to accept higher risk in the quest for higher returns, would be to build a portfolio that tilts toward more volatile stocks, or higher beta stocks, or stocks with higher downside semi-deviation. We might expect these strategies to earn higher portfolio returns, rewarding us for our willingness to bear one of these types of incremental risk. This investment belief anchors our second set of strategies: weighting a portfolio proportional to conventional risk measures such as market beta, volatility, or downside semi-variance of the constituent stocks. The second block of **Exhibit 1**—labeled “High Risk = High Reward”—explores these three strategies and their inverted forms. These strategies all work splendidly, beating the reference cap-weighted benchmark by 2.23% to 2.49% per year.

When we flip the algorithm, now favoring companies with low volatility, low beta, or low downside semi-deviation, we get the expected drop in risk relative to the risk-seeking strategies. Nonetheless, for all three risk-seeking strategies, our returns are even higher when we flip them, to shun risk. The inverted portfolios added between 2.78% and 3.81% per year. These low risk portfolios, as a result, have higher Sharpe Ratios and also higher CAPM alphas.

How can overweighting high risk stocks and overweighting low risk stocks *both* lead to higher returns versus the cap-weighted benchmark? An examination of the FF4 factor decomposition in Table 1 reveals that the key differences between the risk-seeking and risk-averse strategies: the latter have roughly two to three times as large a loading on the value factor and lower loading on the market factor. Net of the value effect and other factor tilts, we are left with annualized FF4 alphas that are statistically similar to zero.

Popular covariance-based strategy indexes vs. their upside-down counterparts

The recent surge in interest in non-price-weighted market indices is a noteworthy development in the evolution of the indexing business. First among many, is the revival of Minimum Variance, with roots dating back to the late 1960s.¹¹ The CAPM Capital Market Line is empirically flatter than theory would predict. Indeed, empirically, it often is downward-sloping: in many markets, we find that low volatility

stocks produce higher returns than high volatility stocks. The Minimum Variance (MinVar) portfolio represents a simple strategy for capturing this anomaly.

Two other new strategies, which lean heavily on the Markowitz mean-variance optimization framework, have been introduced by well-respected quantitative index providers. The “Risk-Efficient” Index assumes, among other things, that stock returns are related to downside semi-variances. The “Maximum Diversification” Index portfolio, on the other hand, assumes a linear relationship between stock returns and volatility. These differ from our earlier exploration of weighting in proportion to volatility or downside semi-variance in the use of an explicit mean-variance optimizer in the portfolio construction. Another covariance-based index strategy is the “Risk Cluster Equal Weight” portfolio, also known as “Diversification Based Index”. The RCEW approach uses equally weighted industry-country clusters, selected on the basis of covariance, to form a portfolio that is, relative to cap-weighting, less concentrated in individual countries and sectors.¹²

Empirically, they all work. In the United States, MinVar outperforms the cap-weighted market by 209 bps per annum. Because of its very low beta and low volatility, the Sharpe ratio is the highest of any strategies that we tested, with the highest statistical significance on the CAPM alpha. However, the excess return is almost fully explained by exposure to FF4 factors, leaving no statistically meaningful FF4 alpha. The other covariance-based strategy indexes also offer historical returns that outperform the cap-weighted market benchmark. As with MinVar, their CAPM alphas are economically large and statistically significant. As with MinVar, the FF4 four-factor model largely explains the excess returns.

In this section, when we invert the strategies, we focus on companies with high rather than low covariance. Again, our inverse strategies deliver outperformance over the cap-weighted benchmark and we observe meaningfully positive CAPM alphas. And, again, positive exposure to value and size explain most of the excess returns, leaving insignificant FF4 alphas.

Favoring stocks with large fundamental scale or earnings growth

Traditional analysts believe that fundamentals matter for stock price valuation: low prices relative to fundamentals suggest undervaluation and better subsequent returns. This fundamental approach anchors the value investing style popularized by Ben Graham in the 1930s and 1940s, which remains influential today. In this section, we test three portfolios weighted by the following fundamental measures: (1) book

value, which tacitly creates a higher book-to-price ratio relative to the cap-weighted benchmark, (2) five-year average of reported earnings, leading to a higher earnings-to-price ratio than the cap-weighted benchmark, and (3) the four-metric composite method described by Arnott, Hsu, and Moore [2005]. All three methods weight stocks drawn from a universe of the 1,000 largest companies in proportion to their financial fundamentals using the method described in AHM 2005.¹³ These portfolios are expected to have a “value tilt,” relative to the cap-weighted market, as the weighting metrics are “value” oriented in nature.

The fourth portfolio in this category is constructed explicitly to have a growth emphasis; it weights stocks proportional to their recent earnings growth. This strategy emphasizes the companies with the strongest recent earnings growth.¹⁴ The Gordon Growth Model suggests that growth in earnings drives stock returns. This has motivated the belief that fast growing companies ought to deliver high returns.

Consistent with the previous sections, Exhibit 1 shows that all of these strategies produce economically meaningful excess returns with no statistically significant FF4 alpha. The first three fundamental-weighted portfolios earn their excess returns from a value tilt, while the earnings growth-weighted strategy outperforms because of its small-cap tilt. It is interesting to observe that we have constructed a growth-oriented portfolio that outperforms the cap-weighted benchmark, unlike most growth strategies, albeit without using cap weights to allocate to the growth stocks. Note that, using FF4 metrics, our growth portfolio actually has a *value tilt*, not a growth tilt, in the U.S. data.

The inverse portfolios should intuitively result in the opposite characteristics and symmetrical results. They do not. Similar to what was observed before, the “upside-down” strategies all win, often by substantial margins, because of positive exposures to value and small-cap. We do observe that a few of these inverted strategies also deliver statistically significant alpha net of the FF4 factor attributes. The positive FF4 alpha is somewhat surprising since these strategies are mechanistic, with no special insights into the subtleties that drive the markets, and thus do not have “skill”. Accordingly, we see two possible interpretations of these significant FF4 alphas. First, they could simply be statistical outliers—after all, 1 in 20 completely random time series will appear to have statistical significance at the 5% level. Alternatively, the outliers could reflect a significant risk factor that is missing from the FF4 model. We leave the exploration of these observations for the future, and welcome others’ investigations into this interesting topic.

Malkiel's blind-folded monkey

In the last section of Exhibit 1, we examine the performance of random portfolios. For those doubting the benefits of active management, the go-to portfolio strategy has been cap-weighted indexing ever since the dawn of the Capital Asset Pricing Model (CAPM). The conventional wisdom generally assumes that the cap-weighted portfolio is the mean-variance efficient, neutral portfolio for investors without stock picking skills. We challenge this premise, by simulating random portfolios managed by Malkiel's dart-throwing monkey for comparison against the cap-weighted benchmark.

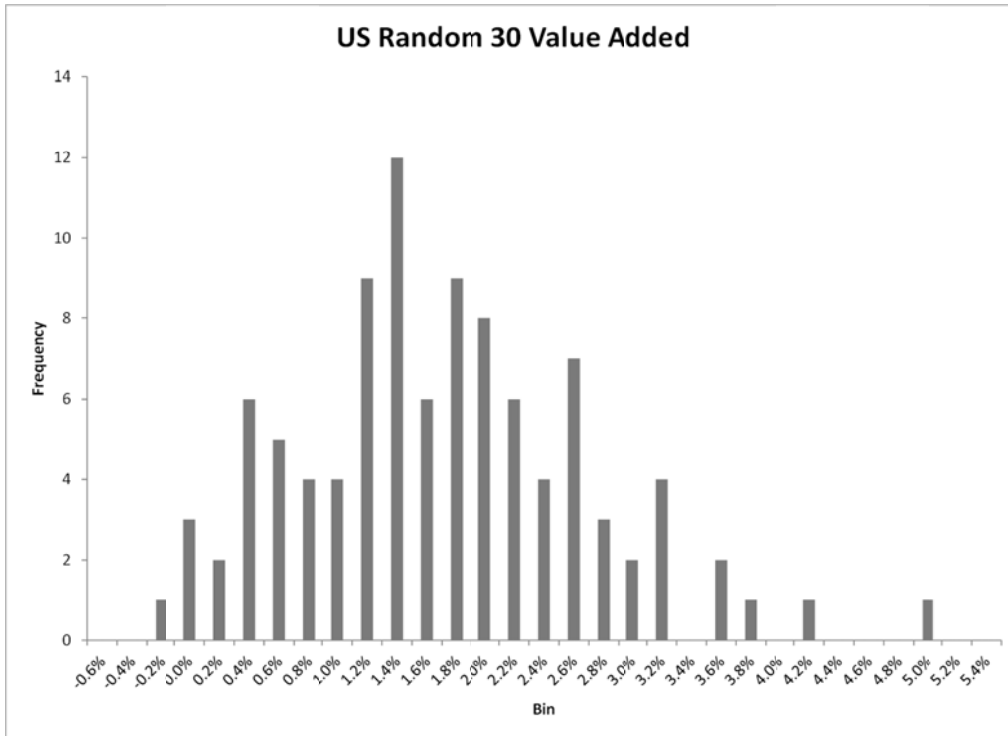
It would be time-consuming and costly to arrange for a monkey to throw darts at the *Wall Street Journal's* stock pages, not to mention tracking down 50 years of archived copies of their stock lists. So, we simulate a dart-throwing monkey, picking a random 30-stock portfolio out of the top 1,000 largest stocks by market capitalization once a year. We then equal-weight the random stock selections to form the portfolio. We repeat the exercise 100 times, and examine both the individual year trials and the average of the trials.

Malkiel surmised that his monkey would perform as well as the market; he was too modest. Our simulated monkey appears to be proficient in security selection, adding an average of 160 bps per year. True, the risk (volatility and beta) and tracking error are large, but we still have a respectable Sharpe ratio and an information ratio that "looks like" skill. **Exhibit 2, Panel A**, shows that the dartboard portfolio matches or beats the cap-weighted portfolio in 99 cases out of the 100 trials. Better still, our monkey has an average CAPM alpha that is economically large and verges on statistical significance¹⁵.

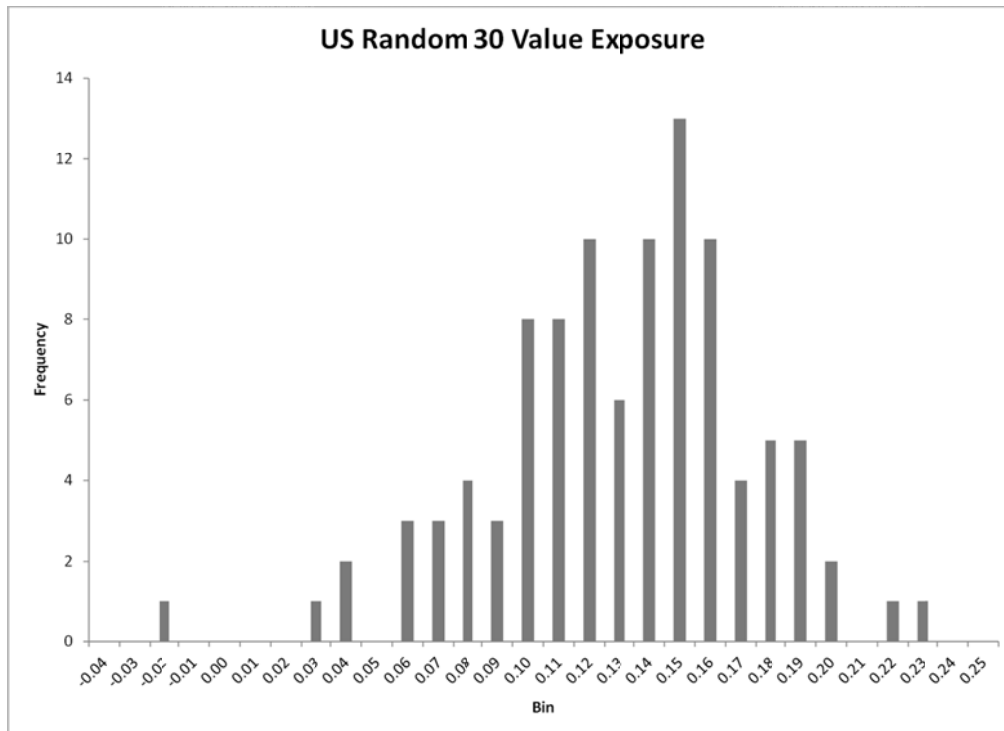
Once again, the FF4 model explains essentially all of the CAPM alpha. Like the other strategy indices that we have examined so far, the monkey is just introducing a size and value tilt. In **Exhibit 2, Panel B**, we can see that the monkey has a value tilt, on average over the 49 years, in 99 of the 100 trials. It should surprise no one that the one trial in which the monkey hits upon a growth tilt is also the one trial that underperforms Cap Weight. The astute observer will note that the average of our 100 monkey-managed portfolios has FF4 factor loadings identical to the Equal Weight portfolio; this is, of course, a trivial convergence result associated with the law of large numbers.

Exhibit 2. Random Strategies: 100 Simulations, United States (1964–2012)

Panel A. Histogram of Outperformance Frequencies



Panel B. Histogram of Value Loading – 100 Simulations US (1964-2012)



Source: Research Affiliates based on CRSP/Compustat data.

Why do these strategies all work?

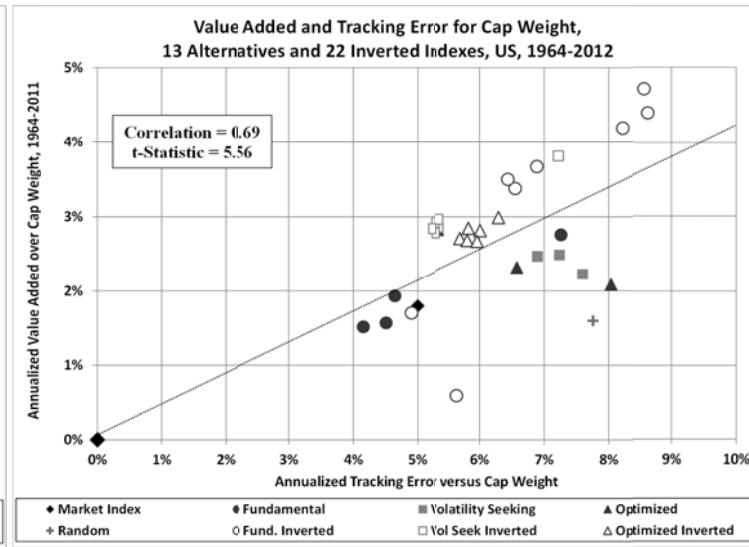
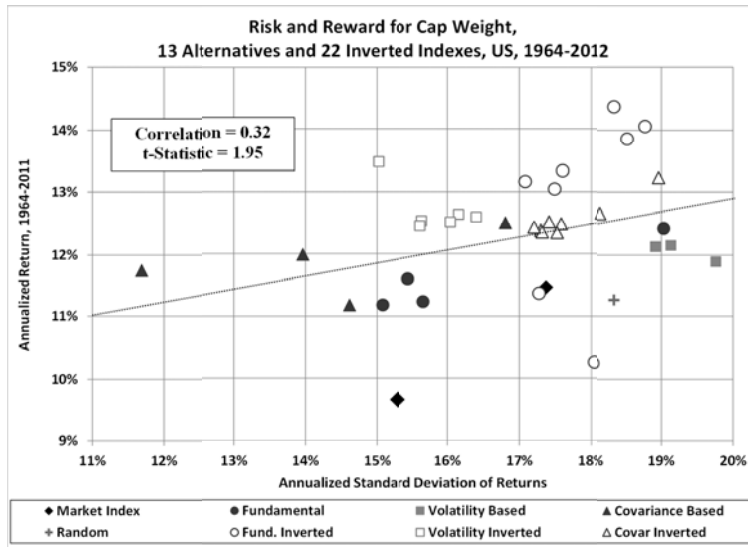
The well-reasoned and carefully crafted strategies tested in this article, which have spawned countless journal articles and white papers, all appear to work remarkably well as shown in the summary statistics at the bottom of Exhibit 1. They only differ by their exposures to market, value, and size, which contributes to their differences in risk and return attributes over time.

When we turn these strategies upside-down, inverting the resulting portfolio weights, we again find a near-perfect pattern of outperformance. Paradoxically, these upside-down strategies generally performed better than the right-side-up strategies that inspired them, with higher returns, higher Sharpe ratios, higher information ratios, and higher CAPM alphas. A clear implication is that the thesis for these alternative non-cap-weight index strategies is not the reason for the outperformance.

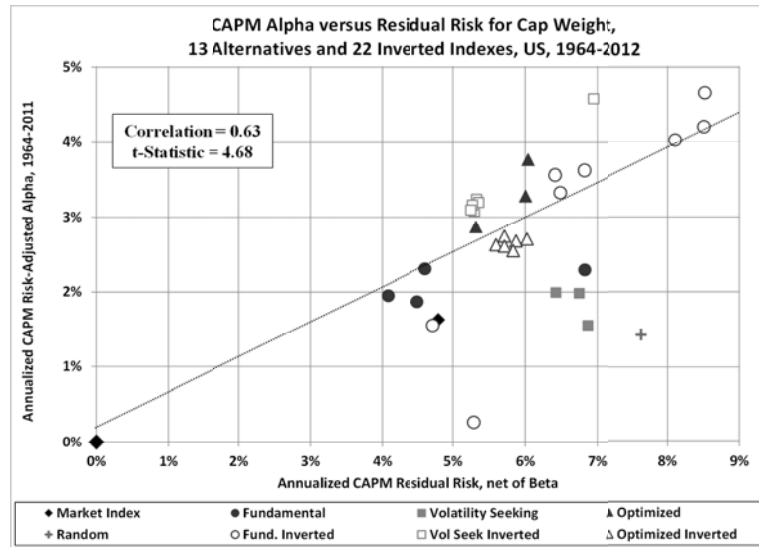
The graphs in **Exhibit 3** provide us with a visual description of the excess return driver. Panel A shows the conventional link between volatility and average returns. Because portfolio volatility is largely determined by its market beta, Panel A would seem to suggest a classic CAPM relationship between beta and return. However, market beta is clearly not the only driver for return given the empirical evidence on value, size and the low volatility effect. Panel B shows the linkage between tracking error and the value added, while Panel C shows a similar linkage between the CAPM residual risk and CAPM alpha, which is conventionally attributed to “skill,” if it’s statistically significant. These two graphs suggest that the entirety of the value-added return shown in Panel A is driven by non-market exposure(s). Panel D shows that, adjusting for the FF4 factor loadings, we are left with a small unexplained alpha and a weak relationship between the FF4 factor model residual and returns. This demonstrates that the FF4 factors are the key drivers of returns. The small unexplained alpha and the weakly positive slope point to a path for future research, which is outside the scope of this paper. There appear to be other priced risk factors (if it’s not skill it presumably must be a risk factor), capable of producing economically meaningful and statistically significant sources of equity returns, which are not captured fully by the FF4 factor model.

Exhibit 3. Performance Characteristics of Market Cap, 14 Strategy Indexes, and 26 Inverses of Same, 1964–2012

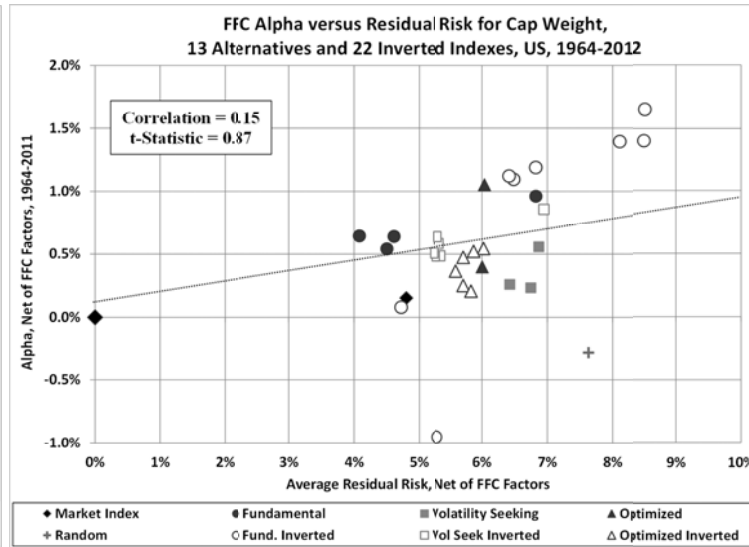
Panel A. Annualized Return versus Standard Deviation of Returns **Panel B. Value Added versus Tracking Error**



Panel C. CAPM Alpha versus Residual Risk



Panel D. FF4 Alpha versus Residual Risk



Source: Research Affiliates based on CRSP/Compustat data.

While the performance of many of these strategies—and their FF4 style attributes—seem markedly similar, especially in their divergence from the less profitable Cap-Weighted strategies, their differences are noteworthy. This is best observed in **Exhibit 4**, which shows the top 10 holdings of a selected roster of these strategies. A casual examination of this table reveals the main problem for the inverse strategies: the top 10 roster is often populated by an array of relatively obscure companies, generally more thinly traded and less liquid than the Cap Weight market leaders. The exceptions are self-evident, and are only found in the original strategies, never their inverse variants.

We draw two important lessons from this research. First, *the investment thesis behind each of these strategies—no matter how thoughtful, intuitive, or compelling—is not the source of the incremental return, or alpha, or information ratio.* The thesis matters little; the resulting value and size tilts are the dominant reason behind the success of these strategies.

Second, a size bias and, more significantly, a value bias exist in almost all of these strategy indexes, whether we engineer for it or not. By comparison, a growth bias seems nearly impossible to find; that's a good thing, given the historical evidence of weak performance for growth-biased portfolios. Indeed, even a portfolio weighted toward stocks with strong historical fundamental growth in earnings exhibits a modest *value* tilt instead of a growth tilt.

In Appendix I, we provide the theoretical explanation for these perplexing empirical observations. Intuitively, any strategy that implicitly weights by a *non-price-based* valuation metric would tend to have a lower price-to-value ratio relative to the cap-weighted index. We shouldn't attribute much, if any, of the success of a strategy to the investment thesis that was the basis of its development.

Further, the inverse portfolios demonstrate that Cap-weighting appears to be surprisingly easy to beat (at least historically). Random portfolios, selected by dart-throwing monkeys, and other inane or bizarre portfolios would evidently do the job.

Exhibit 4. Top 10 Holdings and Weights, Selected Strategies*, Inverse Strategies, and Random Portfolios, (United States, January 1, 2012)

| Market | | High Risk = High Reward | | Optimization-Based | | | | Fundamentals-Based | | | |
|----------------------------------|------|----------------------------------|------|------------------------|------|---------------------------|------|----------------------------|------|-----------------------------|------|
| Capitalization Weighted | | Volatility Weighted | | Minimum Variance | | Risk Cluster Equal Weight | | Fundamental Weighted | | EPS Growth | |
| Exxon Mobil Corp | 3.1% | Human Genome Sciences Inc | 0.5% | Illumina Inc | 3.3% | Altria Group Inc | 5.0% | Exxon Mobil Corp | 3.0% | H S N Inc New | 1.5% |
| Apple Inc | 2.9% | Questcor Pharmaceuticals Inc | 0.4% | Kimberly Clark Corp | 2.9% | Constellation Brands Inc | 5.0% | General Electric Co | 2.5% | Wendys Arbys Group Inc | 1.5% |
| Microsoft Corp | 1.7% | Pier 1 Imports Inc De | 0.4% | Harleysville Group Inc | 2.8% | Reynolds American Inc | 3.4% | Bank Of America Corp | 2.2% | American Water Works Co Inc | 1.2% |
| International Business Machs Cor | 1.7% | American International Group Inc | 0.4% | General Mills Inc | 2.8% | Coca Cola Co | 2.4% | A T & T Inc | 2.1% | Solera Holdings Inc | 1.0% |
| Chevron Corp New | 1.6% | Dollar Thrifty Automotive Grp In | 0.4% | Wal Mart Stores Inc | 2.8% | Lorillard Inc | 2.2% | Wal Mart Stores Inc | 1.9% | Biomarin Pharmaceutical Inc | 1.0% |
| Wal Mart Stores Inc | 1.6% | M G M Mirage | 0.3% | Campbell Soup Co | 2.6% | Loews Corp | 2.1% | Citigroup Inc | 1.9% | Healthsouth Corp | 1.0% |
| General Electric Co | 1.4% | M B I A Inc | 0.3% | Newmont Mining Corp | 2.5% | Mohawk Industries Inc | 1.9% | Chevron Corp New | 1.9% | Ariba Inc | 1.0% |
| Procter & Gamble Co | 1.4% | Genworth Financial Inc | 0.3% | P G & E Corp | 2.4% | Nike Inc | 1.6% | Jpmorgan Chase & Co | 1.7% | Dr Pepper Snapple Group Inc | 1.0% |
| A T & T Inc | 1.4% | Las Vegas Sands Corp | 0.3% | S A I C Inc | 2.4% | Pepsico Inc | 1.6% | Berkshire Hathaway Inc Del | 1.7% | T W Telecom Inc | 0.9% |
| Johnson & Johnson | 1.4% | Medivation Inc | 0.3% | Flowers Foods Inc | 2.2% | Mcdonalds Corp | 1.5% | Pfizer Inc | 1.5% | Domtar Corp | 0.9% |

| Inverse-Ratio of Volatility Weighted | | Inverse-Ratio of Minimum Variance | | Inverse-Ratio of Risk Cluster Equal Weight | | Inverse-Ratio of Fundamental Weighted | | Inverse-Ratio of EPS Growth | |
|--------------------------------------|------|-----------------------------------|--|--|------|---------------------------------------|------|-----------------------------|--|
| Progress Energy Inc | 0.3% | | | P M C Sierra Inc | 0.7% | C B R L Group Inc | 0.3% | | |
| Duke Energy Corp New | 0.3% | | | Netgear Inc | 0.7% | Arris Group Inc | 0.3% | | |
| Southern Co | 0.3% | Not Applicable | | Vishay Intertechnology Inc | 0.6% | John Bean Technologies Corp | 0.2% | Not Applicable | |
| Kimberly Clark Corp | 0.2% | Hundreds of Companies | | Intersil Corp | 0.6% | Geo Group Inc | 0.2% | Hundreds of Companies | |
| General Mills Inc | 0.2% | at the Top of the List | | International Rectifier Corp | 0.6% | W & T Offshore Inc | 0.2% | at the Top of the List | |
| Wisconsin Energy Corp | 0.2% | Have Identical Weight, | | First American Corp Calif | 0.6% | Old Dominion Freight Line Inc | 0.2% | Have Identical Weight, | |
| Nstar | 0.2% | Because of Zero Weight | | Coinstar Inc | 0.6% | Comstock Resources Inc | 0.2% | Because of Zero Weight | |
| Consolidated Edison Inc | 0.2% | in the Original Index | | Itron Inc | 0.6% | Mcmoran Exploration Co | 0.2% | in the Original Index | |
| U G I Corp New | 0.2% | | | Microsemi Corp | 0.6% | Macquarie Infrastructure Co Llc | 0.2% | | |
| X C E L Energy Inc | 0.2% | | | M K S Instruments Inc | 0.6% | Bon Ton Stores Inc | 0.2% | | |

| Inverse-Complement of Volatility Weighted | | Inverse-Complement of Minimum Variance | | Inverse-Complement of Risk Cluster Equal Weight | | Inverse-Complement of Fundamental Weighted | | Inverse-Complement of EPS Growth | |
|---|------|--|--|---|------|--|------|----------------------------------|--|
| Progress Energy Inc | 0.1% | | | P M C Sierra Inc | 0.1% | C B R L Group Inc | 0.1% | | |
| Duke Energy Corp New | 0.1% | | | Netgear Inc | 0.1% | Arris Group Inc | 0.1% | | |
| Southern Co | 0.1% | Not Applicable | | Vishay Intertechnology Inc | 0.1% | John Bean Technologies Corp | 0.1% | Not Applicable | |
| Kimberly Clark Corp | 0.1% | Hundreds of Companies | | Intersil Corp | 0.1% | Geo Group Inc | 0.1% | Hundreds of Companies | |
| General Mills Inc | 0.1% | at the Top of the List | | International Rectifier Corp | 0.1% | W & T Offshore Inc | 0.1% | at the Top of the List | |
| Wisconsin Energy Corp | 0.1% | Have Identical Weight, | | First American Corp Calif | 0.1% | Old Dominion Freight Line Inc | 0.1% | Have Identical Weight, | |
| Nstar | 0.1% | Because of Zero Weight | | Coinstar Inc | 0.1% | Comstock Resources Inc | 0.1% | Because of Zero Weight | |
| Consolidated Edison Inc | 0.1% | in the Original Index | | Itron Inc | 0.1% | Mcmoran Exploration Co | 0.1% | in the Original Index | |
| U G I Corp New | 0.1% | | | Microsemi Corp | 0.1% | Macquarie Infrastructure Co Llc | 0.1% | | |
| X C E L Energy Inc | 0.1% | | | M K S Instruments Inc | 0.1% | Bon Ton Stores Inc | 0.1% | | |

* Note that we exclude equal-weight and random portfolios, neither of which has a well-defined top 10 list. Nor do we include portfolios with a large roster of companies with identical weight at the top of their respective lists.

Source: Research Affiliates based on CRSP/Compustat data.

International Evidence

We extend our analysis to global markets and find that the U.S. results are by no means an aberration. **Exhibit 5** shows the results for the Global Developed World Markets (using the current MSCI definition for our roster of countries), from 1991–2012.¹⁶ With only one exception, all these global strategies historically added value. And, with only one exception, the inverted strategies also add value. The CAPM alphas for the strategies are almost all positive, many showing statistical significance. For 18 of the 22 inverted strategies, results are better than the underlying strategy. And, the FF4 alphas in the global arena are generally stronger, both in economic terms and in statistical significance, than for the United States, despite a shorter history. Let the quest for the missing risk factor(s) begin!

Summary

Many sensible investment beliefs, when translated into portfolio weighting strategies, result in outperformance against the cap-weighted benchmark index. But, so do the arguably nonsensical inverses of those weighting strategies. This paradoxical empirical result, which is observed in a large array of long-only strategies globally, is a consequence of the fact that seemingly unrelated and non-“value”/non-“small-cap” based strategies often have unintended and almost unavoidable value and small-cap tilts, as do their inverse strategies.

The resulting factor tilts are the primary sources of the outperformance, rather than the underlying investment beliefs. Even Malkiel’s blind-folded monkey throwing darts at the *Wall Street Journal* would produce a portfolio strategy with a value and size bias that would have outperformed historically. Our empirical results support an assertion that value and size arise naturally in non-price-weighted strategies and constitute the main source of their return advantage.

What are we to make of the result that popular strategy indexes, when inverted, produce even better outperformance? It, perhaps, behooves investors to emphasize more the FF4 factor based analysis when analyzing investment philosophies. When random portfolios and irrational investment strategies all lead to outperformance, a simple outperformance measure becomes an unreliable gauge for skill.

Exhibit 5. Performance Summary, Strategies, Inverse Strategies, and Random Portfolios, (Global Developed Markets, 1991–2012)

| Strategy | Return | Standard Deviation | Sharpe Ratio | Value Added | Tracking Error | Information Ratio | CAPM Alpha | CAPM Beta | Alpha t-Stat | Annual FFC Alpha | Alpha t-stat | Market Exposure | Size Exposure | Value Exposure | Momentum Exposure | FFC Residual | CAPM Residual |
|--|--------------|--------------------|--------------|--------------|----------------|-------------------|--------------|-------------|--------------|------------------|--------------|-----------------|---------------|----------------|-------------------|--------------|---------------|
| Global Cap Weighted | 7.15% | 15.15% | 0.26 | 0.00% | 0.00% | 0.00 | 0.00% | 1.00 | 0.00 | 0.00% | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00% | 0.00% |
| Equal Weight | 8.36% | 15.45% | 0.34 | 1.21% | 2.71% | 0.45 | 1.32% | 1.00 | 2.04 | 0.28% | 0.65 | 1.02 | 0.25 | 0.15 | -0.02 | 1.86% | 2.80% |
| Volatility Weighted | 7.86% | 16.89% | 0.28 | 0.71% | 3.84% | 0.19 | 0.51% | 1.08 | 0.55 | 0.12% | 0.20 | 1.10 | 0.31 | 0.13 | -0.06 | 2.71% | 3.67% |
| Market Beta Weighted | 6.58% | 18.81% | 0.18 | -0.57% | 5.81% | -0.10 | -1.24% | 1.20 | -0.89 | -0.13% | -0.13 | 1.19 | 0.37 | 0.03 | -0.15 | 4.00% | 5.13% |
| Downside Semi-Deviation Weighted | 8.29% | 16.78% | 0.31 | 1.14% | 3.88% | 0.29 | 0.97% | 1.07 | 1.04 | 0.55% | 0.83 | 1.09 | 0.29 | 0.15 | -0.07 | 2.82% | 3.76% |
| Inverse-Ratio of Volatility Weighted | 9.32% | 13.94% | 0.44 | 2.17% | 4.11% | 0.53 | 2.73% | 0.89 | 2.77 | 0.77% | 1.28 | 0.92 | 0.13 | 0.34 | -0.04 | 2.55% | 3.91% |
| Inverse-Complement of Volatility Weighted | 8.99% | 14.81% | 0.40 | 1.84% | 3.42% | 0.54 | 2.16% | 0.95 | 2.63 | 0.69% | 1.28 | 0.98 | 0.19 | 0.27 | -0.05 | 2.28% | 3.49% |
| Inverse-Ratio of Market Beta Weighted | 9.44% | 12.34% | 0.51 | 2.29% | 6.85% | 0.33 | 3.49% | 0.72 | 2.12 | 0.66% | 0.64 | 0.77 | 0.01 | 0.44 | 0.01 | 4.35% | 5.65% |
| Inverse-Complement of Market Beta Weighted | 9.31% | 14.31% | 0.43 | 2.16% | 3.84% | 0.56 | 2.65% | 0.91 | 2.87 | 0.71% | 1.12 | 0.94 | 0.17 | 0.30 | -0.01 | 2.69% | 3.74% |
| Inverse-Ratio of Downside Semi-Deviation Weighted | 9.11% | 13.89% | 0.43 | 1.96% | 4.08% | 0.48 | 2.53% | 0.88 | 2.58 | 0.54% | 0.90 | 0.92 | 0.14 | 0.33 | -0.03 | 2.54% | 3.86% |
| Inverse-Complement of Downside Semi-Deviation Weighted | 8.83% | 14.41% | 0.40 | 1.68% | 3.71% | 0.45 | 2.11% | 0.92 | 2.37 | 0.36% | 0.62 | 0.95 | 0.17 | 0.30 | -0.04 | 2.42% | 3.68% |
| Minimum Variance | 8.40% | 9.89% | 0.53 | 1.25% | 9.65% | 0.13 | 3.20% | 0.53 | 1.38 | 1.73% | 1.33 | 0.55 | 0.02 | 0.30 | -0.06 | 5.49% | 6.15% |
| Maximum Diversification | 7.14% | 11.33% | 0.35 | 0.00% | 9.09% | 0.00 | 1.59% | 0.62 | 0.73 | 0.12% | 0.08 | 0.65 | 0.11 | 0.24 | 0.01 | 6.52% | 6.77% |
| Risk-Efficient ($\lambda=2$) | 9.00% | 14.82% | 0.40 | 1.85% | 3.47% | 0.53 | 2.17% | 0.95 | 2.61 | 0.53% | 0.93 | 0.98 | 0.19 | 0.28 | -0.03 | 2.40% | 3.53% |
| Risk Cluster Equal Weight | 9.48% | 15.90% | 0.40 | 2.33% | 6.54% | 0.36 | 2.63% | 0.95 | 1.68 | 0.97% | 0.66 | 1.00 | 0.25 | 0.21 | 0.08 | 6.25% | 6.67% |
| Inverse-Ratio of Minimum Variance | 8.70% | 16.22% | 0.34 | 1.55% | 3.46% | 0.45 | 1.50% | 1.04 | 1.80 | 0.42% | 0.76 | 1.07 | 0.24 | 0.23 | -0.05 | 2.36% | 3.50% |
| Inverse-Complement of Minimum Variance | 8.77% | 15.50% | 0.36 | 1.62% | 3.32% | 0.49 | 1.75% | 0.99 | 2.20 | 0.47% | 0.88 | 1.02 | 0.22 | 0.25 | -0.05 | 2.28% | 3.44% |
| Inverse-Ratio of Maximum Diversification | 8.90% | 15.86% | 0.36 | 1.75% | 3.67% | 0.48 | 1.81% | 1.01 | 2.06 | 0.50% | 0.88 | 1.04 | 0.21 | 0.29 | -0.07 | 2.41% | 3.80% |
| Inverse-Complement of Maximum Diversification | 8.80% | 15.35% | 0.37 | 1.65% | 3.38% | 0.49 | 1.83% | 0.98 | 2.26 | 0.49% | 0.91 | 1.01 | 0.21 | 0.26 | -0.05 | 2.30% | 3.50% |
| Inverse-Ratio of Risk-Efficient ($\lambda=2$) | 8.55% | 15.46% | 0.35 | 1.40% | 3.52% | 0.40 | 1.56% | 0.99 | 1.85 | 0.44% | 0.75 | 1.01 | 0.22 | 0.25 | -0.06 | 2.48% | 3.65% |
| Inverse-Complement of Risk-Efficient ($\lambda=2$) | 8.51% | 15.68% | 0.34 | 1.37% | 3.59% | 0.38 | 1.47% | 1.00 | 1.71 | 0.45% | 0.75 | 1.02 | 0.23 | 0.24 | -0.07 | 2.57% | 3.72% |
| Inverse-Ratio of RCEW | 9.44% | 16.70% | 0.38 | 2.29% | 6.53% | 0.35 | 2.34% | 1.02 | 1.50 | 0.63% | 0.42 | 1.05 | 0.14 | 0.28 | 0.02 | 6.41% | 6.78% |
| Inverse-Complement of RCEW | 8.74% | 15.22% | 0.37 | 1.59% | 3.41% | 0.47 | 1.80% | 0.97 | 2.21 | 0.47% | 0.86 | 1.00 | 0.21 | 0.26 | -0.05 | 2.32% | 3.52% |
| Book Value Weighted | 9.50% | 16.09% | 0.40 | 2.35% | 4.78% | 0.49 | 2.47% | 1.00 | 2.15 | 1.31% | 2.22 | 1.02 | 0.09 | 0.40 | -0.12 | 2.50% | 4.79% |
| 5yr avg Earnings Weighted | 11.20% | 15.28% | 0.51 | 3.83% | 5.01% | 0.76 | 3.65% | 0.95 | 3.04 | 2.36% | 3.28 | 0.97 | -0.01 | 0.39 | -0.09 | 3.05% | 4.97% |
| Fundamental Weighted | 11.00% | 15.33% | 0.49 | 3.63% | 5.06% | 0.72 | 3.43% | 0.96 | 2.82 | 1.93% | 2.98 | 0.98 | 0.09 | 0.43 | -0.11 | 2.74% | 5.03% |
| Earnings Growth Weighted | 8.83% | 17.06% | 0.33 | 1.68% | 4.19% | 0.40 | 1.37% | 1.11 | 1.36 | 1.55% | 1.91 | 1.11 | 0.27 | -0.02 | -0.04 | 3.44% | 3.99% |
| Inverse-Ratio of Book Value Weighted | 10.60% | 15.51% | 0.48 | 3.45% | 5.65% | 0.61 | 3.76% | 0.95 | 2.78 | 1.94% | 2.60 | 0.98 | 0.33 | 0.46 | -0.13 | 3.16% | 5.76% |
| Inverse-Complement of Book Value Weighted | 10.51% | 15.60% | 0.47 | 3.37% | 5.30% | 0.64 | 3.64% | 0.96 | 2.86 | 1.95% | 2.90 | 0.99 | 0.26 | 0.45 | -0.13 | 2.84% | 5.41% |
| Inverse-Ratio of 5yr avg Earnings Weighted | 12.45% | 15.40% | 0.58 | 5.08% | 6.12% | 0.83 | 4.82% | 0.94 | 3.29 | 2.70% | 3.28 | 0.98 | 0.29 | 0.50 | -0.12 | 3.49% | 6.06% |
| Inverse-Complement of 5yr avg Earnings Weighted | 12.40% | 15.35% | 0.58 | 5.03% | 5.70% | 0.88 | 4.77% | 0.94 | 3.49 | 2.79% | 3.63 | 0.98 | 0.21 | 0.48 | -0.11 | 3.25% | 5.64% |
| Inverse-Ratio of Fundamental Weighted | 12.53% | 15.67% | 0.58 | 5.16% | 6.41% | 0.80 | 4.73% | 0.95 | 3.08 | 2.81% | 3.44 | 0.99 | 0.35 | 0.51 | -0.15 | 3.45% | 6.37% |
| Inverse-Complement of Fundamental Weighted | 12.32% | 15.50% | 0.57 | 4.95% | 5.91% | 0.84 | 4.56% | 0.95 | 3.22 | 2.74% | 3.70 | 0.98 | 0.28 | 0.49 | -0.13 | 3.14% | 5.87% |
| Inverse-Ratio of Earnings Growth Weighted | 6.60% | 15.92% | 0.22 | -0.55% | 4.51% | -0.12 | -0.41% | 0.99 | -0.38 | -1.20% | -1.57 | 1.02 | 0.43 | 0.06 | 0.02 | 3.25% | 4.57% |
| Inverse-Complement of Earnings Growth Weighted | 8.36% | 15.24% | 0.34 | 1.22% | 2.73% | 0.45 | 1.39% | 0.99 | 2.12 | 0.23% | 0.54 | 1.01 | 0.25 | 0.17 | -0.02 | 1.81% | 2.81% |
| Average of 100 Malkiel's Monkey Portfolios | 8.12% | 16.36% | 0.31 | 0.97% | 6.35% | 0.16 | 1.10% | 1.00 | 0.72 | 0.15% | 0.10 | 1.02 | 0.23 | 0.18 | -0.03 | 5.92% | 6.34% |
| Average for Non-Cap-Weight Strategies, excl. Inverses | 8.75% | 15.38% | 0.37 | 1.57% | 5.41% | 0.34 | 1.78% | 0.96 | 1.48 | 0.88% | 1.15 | 0.98 | 0.19 | 0.22 | -0.05 | 3.82% | 4.89% |
| Average for All Inverse-Ratio Strategies | 9.60% | 15.17% | 0.43 | 2.41% | 4.99% | 0.47 | 2.62% | 0.94 | 2.13 | 0.93% | 1.22 | 0.98 | 0.23 | 0.34 | -0.06 | 3.31% | 4.90% |
| Average for All Inverse-Complement Strategies | 9.60% | 15.18% | 0.42 | 2.41% | 4.03% | 0.56 | 2.56% | 0.96 | 2.54 | 1.03% | 1.56 | 0.99 | 0.22 | 0.32 | -0.06 | 2.54% | 4.07% |

Source: Research Affiliates based on Worldscope/Datastream data.

Exhibit 6. Global Strategies Performance Summary (1991–2012)*

| Country | Strategy | Cap Weighted | Volatility Weighted | Market Beta Weighted | Inverse Ratio of Volatility Weighted | Inverse Complement of Volatility Weighted | Inverse Ratio of Market Beta Weighted | Inverse Complement of Market Beta Weighted | Minimum Variance | Minimum Diversification | Inverse Ratio of Minimum Variance | Inverse Complement of Minimum Variance | Inverse Ratio of Minimum Diversification | Inverse Complement of Minimum Diversification | Book Weighted | Inverse Ratio of Book Weighted | Inverse Complement of Book Weighted |
|-----------|-----------------------|--------------|---------------------|----------------------|--------------------------------------|---|---------------------------------------|--|------------------|-------------------------|-----------------------------------|--|--|---|---------------|--------------------------------|-------------------------------------|
| Australia | Return | 12.4% | 16.7% | 16.4% | 15.3% | 16.6% | 17.6% | 16.6% | 17.4% | 23.8% | 15.5% | 16.2% | 14.4% | 15.5% | 14.3% | 14.2% | 13.9% |
| | Volatility | 20.9% | 27.4% | 26.4% | 20.8% | 22.2% | 22.1% | 22.3% | 19.2% | 25.9% | 28.8% | 24.3% | 22.3% | 22.8% | 21.2% | 23.4% | 22.2% |
| | Value Added | | 4.3% | 4.0% | 2.9% | 4.2% | 5.1% | 4.2% | 5.0% | 11.3% | 3.0% | 3.7% | 1.9% | 3.1% | 1.9% | 1.8% | 1.5% |
| | 4-factor Alpha | | 5.7% | 4.8% | 3.8% | 4.9% | 6.4% | 5.0% | 5.6% | 12.1% | 5.4% | 4.7% | 3.0% | 4.0% | 2.0% | 3.7% | 2.7% |
| | 4-factor Alpha t-stat | | 1.76 | 2.03 | 3.04 | 3.14 | 2.84 | 2.76 | 3.34 | 3.36 | 1.48 | 2.24 | 2.17 | 2.52 | 2.46 | 2.09 | 1.93 |
| | Size Exposure | | 0.48 | 0.32 | 0.20 | 0.26 | 0.31 | 0.29 | 0.12 | 0.41 | 0.48 | 0.35 | 0.25 | 0.29 | -0.02 | 0.40 | 0.25 |
| | Value Exposure | | -0.05 | -0.05 | -0.01 | -0.01 | -0.02 | -0.02 | 0.02 | -0.04 | -0.04 | -0.02 | -0.03 | -0.03 | 0.01 | -0.02 | -0.01 |
| Canada | Return | 10.5% | 10.5% | 10.3% | 12.6% | 12.3% | 10.9% | 12.1% | 11.7% | 10.5% | 10.8% | 11.6% | 12.6% | 12.1% | 11.8% | 11.8% | 12.0% |
| | Volatility | 19.3% | 20.7% | 20.6% | 17.1% | 17.7% | 19.2% | 18.0% | 15.1% | 17.7% | 23.5% | 20.2% | 18.9% | 18.9% | 18.2% | 18.4% | 18.2% |
| | Value Added | | -0.1% | -0.2% | 2.0% | 1.8% | 0.3% | 1.5% | 1.1% | 0.0% | 0.2% | 1.1% | 2.1% | 1.6% | 1.3% | 1.3% | 1.4% |
| | 4-factor Alpha | | -0.5% | 0.5% | 1.6% | 1.3% | -0.6% | 0.8% | 1.6% | 0.0% | -0.8% | 0.3% | 1.4% | 0.9% | 1.5% | 1.2% | 1.4% |
| | 4-factor Alpha t-stat | | -0.32 | 0.36 | 1.40 | 1.08 | -0.30 | 0.59 | 1.10 | 0.01 | -0.39 | 0.21 | 0.98 | 0.75 | 1.77 | 0.80 | 1.02 |
| | Size Exposure | | 0.20 | 0.18 | 0.07 | 0.08 | 0.04 | 0.10 | 0.08 | 0.18 | 0.14 | 0.14 | 0.04 | 0.11 | -0.01 | 0.16 | 0.11 |
| | Value Exposure | | 0.15 | 0.04 | 0.31 | 0.30 | 0.43 | 0.33 | 0.33 | 0.20 | 0.22 | 0.22 | 0.35 | 0.25 | 0.25 | 0.35 | 0.35 |
| France | Return | 8.7% | 10.2% | 9.8% | 11.3% | 11.3% | 11.9% | 11.1% | 12.3% | 10.8% | 10.9% | 10.3% | 10.6% | 10.6% | 9.5% | 12.8% | 11.7% |
| | Volatility | 20.0% | 21.2% | 22.2% | 19.5% | 19.6% | 18.8% | 19.2% | 16.8% | 18.6% | 23.3% | 21.9% | 21.7% | 21.6% | 21.5% | 20.4% | 20.6% |
| | Value Added | | 1.5% | 1.1% | 2.7% | 2.7% | 3.2% | 2.5% | 3.7% | 2.5% | 2.2% | 1.6% | 1.9% | 1.9% | 0.8% | 4.1% | 3.1% |
| | 4-factor Alpha | | 1.8% | 2.2% | 2.2% | 2.3% | 2.2% | 1.6% | 3.0% | 1.7% | 3.0% | 1.8% | 2.2% | 1.8% | 0.9% | 3.9% | 3.0% |
| | 4-factor Alpha t-stat | | 1.62 | 1.96 | 2.35 | 2.40 | 1.99 | 1.57 | 2.24 | 1.34 | 2.15 | 1.65 | 1.89 | 1.69 | 1.26 | 2.90 | 2.72 |
| | Size Exposure | | 0.42 | 0.40 | 0.35 | 0.35 | 0.38 | 0.38 | 0.31 | 0.44 | 0.44 | 0.41 | 0.32 | 0.38 | 0.09 | 0.60 | 0.47 |
| | Value Exposure | | 0.16 | 0.11 | 0.22 | 0.20 | 0.27 | 0.25 | 0.23 | 0.16 | 0.16 | 0.18 | 0.24 | 0.23 | 0.29 | 0.30 | 0.29 |
| Japan | Return | 0.3% | 0.9% | 0.6% | 2.2% | 1.9% | 1.5% | 1.7% | 1.2% | -0.6% | 1.5% | 1.7% | 1.9% | 1.8% | 3.4% | 3.9% | 3.8% |
| | Volatility | 20.1% | 21.0% | 21.9% | 18.8% | 19.1% | 19.1% | 19.3% | 15.4% | 17.8% | 20.8% | 20.2% | 20.3% | 20.0% | 20.0% | 20.5% | 20.3% |
| | Value Added | | 0.6% | 0.3% | 1.9% | 1.7% | 1.3% | 1.4% | 0.9% | -0.9% | 1.2% | 1.4% | 1.6% | 1.5% | 3.2% | 3.6% | 3.5% |
| | 4-factor Alpha | | -0.2% | 0.4% | 0.1% | 0.0% | -0.6% | -0.2% | -1.8% | -2.5% | 0.1% | 0.1% | 0.3% | 0.2% | 1.3% | 1.5% | 1.5% |
| | 4-factor Alpha t-stat | | -0.23 | 0.34 | 0.08 | 0.04 | -0.39 | -0.22 | -1.15 | -1.67 | 0.18 | 0.12 | 0.33 | 0.20 | 2.52 | 1.71 | 1.88 |
| | Size Exposure | | 0.27 | 0.21 | 0.23 | 0.24 | 0.31 | 0.26 | 0.15 | 0.20 | 0.27 | 0.26 | 0.25 | 0.25 | 0.05 | 0.39 | 0.30 |
| | Value Exposure | | 0.20 | 0.10 | 0.29 | 0.26 | 0.29 | 0.26 | 0.29 | 0.20 | 0.24 | 0.25 | 0.26 | 0.25 | 0.33 | 0.39 | 0.38 |
| UK | Return | 8.0% | 8.7% | 7.5% | 9.2% | 9.1% | 10.1% | 9.4% | 9.5% | 10.1% | 9.0% | 8.8% | 8.5% | 9.0% | 9.5% | 10.0% | 10.4% |
| | Volatility | 16.4% | 18.3% | 20.0% | 16.3% | 16.2% | 15.5% | 16.0% | 14.9% | 15.4% | 20.4% | 18.1% | 18.9% | 18.1% | 18.9% | 19.1% | 18.6% |
| | Value Added | | 0.7% | -0.5% | 1.2% | 1.1% | 2.1% | 1.4% | 1.5% | 2.1% | 1.0% | 0.8% | 0.5% | 1.0% | 1.5% | 2.0% | 2.4% |
| | 4-factor Alpha | | 1.9% | 2.6% | 1.4% | 1.2% | 1.1% | 1.1% | 1.4% | 1.7% | 2.5% | 1.7% | 2.0% | 1.9% | 1.6% | 2.2% | 2.4% |
| | 4-factor Alpha t-stat | | 1.65 | 1.78 | 1.41 | 1.24 | 0.97 | 1.20 | 0.96 | 1.49 | 1.55 | 1.56 | 1.58 | 1.75 | 1.69 | 1.56 | 2.03 |
| | Size Exposure | | 0.30 | 0.25 | 0.21 | 0.21 | 0.24 | 0.24 | 0.20 | 0.21 | 0.23 | 0.26 | 0.25 | 0.26 | 0.08 | 0.41 | 0.31 |
| | Value Exposure | | -0.03 | -0.13 | 0.09 | 0.10 | 0.15 | 0.12 | 0.10 | 0.10 | -0.03 | 0.03 | 0.02 | 0.04 | 0.31 | 0.35 | 0.34 |
| Global | Return | 7.1% | 7.9% | 6.6% | 9.3% | 9.0% | 9.4% | 9.3% | 8.4% | 7.1% | 8.7% | 8.8% | 8.9% | 8.8% | 9.5% | 10.6% | 10.5% |
| | Volatility | 15.1% | 16.9% | 18.8% | 13.9% | 14.8% | 12.3% | 14.3% | 9.9% | 11.3% | 16.2% | 15.5% | 15.9% | 15.4% | 16.1% | 15.5% | 15.6% |
| | Value Added | | 0.7% | -0.6% | 2.2% | 1.8% | 2.3% | 2.2% | 1.2% | 0.0% | 1.5% | 1.6% | 1.7% | 1.6% | 2.4% | 3.4% | 3.4% |
| | 4-factor Alpha | | 0.1% | -0.1% | 0.8% | 0.7% | 0.7% | 0.7% | 1.7% | 0.1% | 0.4% | 0.5% | 0.5% | 0.5% | 1.3% | 1.9% | 2.0% |
| | 4-factor Alpha t-stat | | 0.20 | -0.13 | 1.28 | 1.28 | 0.64 | 1.12 | 1.33 | 0.08 | 0.76 | 0.88 | 0.88 | 0.91 | 2.22 | 2.60 | 2.90 |
| | Size Exposure | | 0.31 | 0.37 | 0.13 | 0.19 | 0.01 | 0.17 | 0.02 | 0.11 | 0.24 | 0.22 | 0.21 | 0.21 | 0.09 | 0.33 | 0.26 |
| | Value Exposure | | 0.13 | 0.03 | 0.34 | 0.27 | 0.44 | 0.30 | 0.30 | 0.24 | 0.23 | 0.25 | 0.29 | 0.26 | 0.40 | 0.46 | 0.45 |

*Due to space limitations we report only a fraction of the inverse strategies for the international markets. Omitted simulations display similar results and are available upon request.

Source: Research Affiliates based on Worldscope/Datastream data.

Exhibit 7. Global Random Strategies Performance Summary (1991–2012)

| Country | Strategy | Return | Volatility | Sharpe Ratio | Value Added | Tracking Error | Information Ratio | Outperformed out of 100 |
|-----------|-----------------------|--------|------------|--------------|-------------|----------------|-------------------|-------------------------|
| Australia | Cap Weighted | 12.41% | 20.90% | 0.44 | | | | |
| | 100 Portfolio Avg | 12.83% | 22.62% | 0.43 | 0.42% | 7.84% | 0.05 | 68 |
| | 100 Portfolio Std Dev | 1.36% | 0.52% | 0.06 | 1.36% | 0.48% | 0.18 | |
| Canada | Cap Weighted | 10.55% | 19.32% | 0.38 | | | | |
| | 100 Portfolio Avg | 11.82% | 19.05% | 0.46 | 1.27% | 7.31% | 0.17 | 88 |
| | 100 Portfolio Std Dev | 1.11% | 0.51% | 0.06 | 1.11% | 0.47% | 0.15 | |
| France | Cap Weighted | 8.67% | 19.98% | 0.28 | | | | |
| | 100 Portfolio Avg | 10.72% | 20.81% | 0.37 | 2.05% | 6.00% | 0.34 | 100 |
| | 100 Portfolio Std Dev | 0.85% | 0.38% | 0.04 | 0.85% | 0.32% | 0.14 | |
| Japan | Cap Weighted | 0.29% | 20.10% | -0.14 | | | | |
| | 100 Portfolio Avg | 1.71% | 20.63% | -0.07 | 1.43% | 7.37% | 0.20 | 88 |
| | 100 Portfolio Std Dev | 1.51% | 0.56% | 0.07 | 1.51% | 0.49% | 0.21 | |
| UK | Cap Weighted | 7.99% | 16.42% | 0.30 | | | | |
| | 100 Portfolio Avg | 9.12% | 17.66% | 0.34 | 1.12% | 5.91% | 0.19 | 92 |
| | 100 Portfolio Std Dev | 0.82% | 0.40% | 0.05 | 0.82% | 0.36% | 0.14 | |
| Global | Cap Weighted | 7.15% | 15.15% | 0.27 | | | | |
| | 100 Portfolio Avg | 8.12% | 16.36% | 0.31 | 0.97% | 6.35% | 0.16 | 76 |
| | 100 Portfolio Std Dev | 1.26% | 0.62% | 0.08 | 1.26% | 0.79% | 0.20 | |

Source: Research Affiliates based on Worldscope/Datastream data.

We omit from this study the discussion of transaction costs and investment capacity. This is done for simplicity, given our purpose. At the same time, the costs and capacity differences between strategies can make a significant difference for investors who are interested in assessing the true investment benefits of these strategies. Given that both sensible and senseless strategies outperform for the same reasons (value and small-cap tilts), potential investors would do well to base much of their decision on the comparison of implementation costs associated with turnover and market price impact.¹⁷

References

- Amenc, Noël, Felix Goltz, Lionel Martellini, and Patrice Retkowsky. 2010. "Efficient Indexation: An Alternative to Cap-Weighted Indices." *EDHEC-Risk Institution Publication* (January).
- Arnott, Robert D., and Jason C. Hsu. 2008. "Noise, CAPM and the Size and Value Effects." *Journal of Investment Management*, vol. 6, no. 1 (First Quarter):1–11.
- Arnott, Robert D., Jason C. Hsu, Jun Liu, and Harry Markowitz. 2011. "Can Noise Create the Size and Value Effects?" Working Paper, University of California at San Diego and Research Affiliates.
- Arnott, Robert D., Jason C. Hsu, and Philip Moore. 2005. "Fundamental Indexation." *Financial Analysts Journal*, vol. 61, no. 2 (March/April):83–99.
- Asness, Cliff S. 1994. "Variables that Explain Stock Returns." Ph.D. Dissertation, University of Chicago.
- Banko, John C., Mitchell Conover, and Gerald R. Jensen. 2006. "The Relationship between the Value Effect and Industry Affiliation." *Journal of Business*, vol. 79, no. 5 (September):2595–2616.
- Berk, Jonathan B. 1997. "Does Size Really Matter?" *Financial Analysts Journal*, vol. 53, no. 5 (September/October):12–18.
- Carhart, Mark M. 1997. "On Persistence in Mutual Fund Performance." *Journal of Finance*, vol. 52, no. 1 (March):57–82.
- Choueifaty, Yves, and Yves Coignard. 2008. "Toward Maximum Diversification." *Journal of Portfolio Management*, vol. 35, no. 1 (Fall):40–51.
- Chow, Tzee-man, Jason Hsu, Vitali Kalesnik and Bryce Little. 2011. "A Survey of Alternative Equity Index Strategies." *Financial Analysts Journal*, vol. 67, no. 5 (September/October):37–57.

Clarke, Roger G., Harindra de Silva, and Steven Thorley. 2006. "Minimum-Variance Portfolios in the U.S. Equity Market." *Journal of Portfolio Management*, vol. 33, no. 1 (Fall):10–24.

Clare, Andrew, Motson, Nick and Thomas, Steve. 2013. "An Evaluation of Alternative Equity Indices—Part 1: Heuristic and Optimised Weighting Schemes." Cass Consulting. (March).

Clare, Andrew, Motson, Nick and Thomas, Steve. 2013. "An Evaluation of Alternative Equity Indices—Part 2: Fundamental Weighting Schemes." Cass Consulting. (March).

Cohen, Randolph B., and Christopher K. Polk. 1998. "The Impact of Industry Factors in Asset-Pricing Tests." Working paper, Kellogg Graduate School of Management, Northwestern University.

Graham, Jeffrey. 2012. "Comment on the Theoretical and Empirical Evidence of Fundamental Indexing," *Journal of Investment Management*, vol. 10, no. 1 (First Quarter).

Haugen, Robert A., and A. James Heins. 1975. "Risk and the Rate of Return on Financial Assets: Some Old Wine in New Bottles." *Journal of Financial and Quantitative Analysis*, vol. 10, no 5, (December):775–784.

Hsu, Jason C. 2006. "Cap-Weighted Portfolios Are Sub-Optimal Portfolios." *Journal of Investment Management*, vol. 4, no. 3 (Third Quarter):1–10.

Malkiel, Burton G. 2007. *A Random Walk Down Wall Street*. New York: W.W. Norton & Company, Inc.

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² In his bestselling book *A Random Walk Down Wall Street*, Burton Malkiel claimed that "a blindfolded monkey throwing darts at a newspaper's financial pages could select a portfolio that would do just as well as one carefully selected by experts." who, he believed, would on average produce results that are no better than the cap-weighted benchmark. The implicit assumption here is that both monkeys and equity portfolio managers have no skills when prices are random walks and therefore would perform no better than the cap-weighted benchmark. As it turns out, Dr. Malkiel's assessment of his monkey was too modest; in empirical testing, the monkey reliably outperforms, at least before transaction costs

³ This result is unsurprising. A sufficiently large random portfolio converges on equal-weight, which has well documented and well understood value added over corresponding cap-weighting of the same names.

⁴ The aim of this paper is neither to recommend for or against any particular strategy index. Essentially all of the strategies examined in this paper and their inverses have provided highly profitable factor tilts; some of them also have attractively low turnover, vast capacity, and appealing core-like portfolio composition, making them interesting investment options. There is value in strategies that give well-constructed access to value and small-cap exposure.

⁵ Our research draws on the work of Chow, Hsu, Kalesnik, and Little [2011] who find that popular alternative equity indexing strategies outperform due largely to their value and size exposures.

⁶ In the Inverse Ratio strategies, for stocks with a weight of “0” in the original portfolio, the inverted “1/w” weight is set to the inverse of the lowest non-zero weight to avoid singularity. Note that when a strategy sets most of the 1,000 stocks to zero weight, the inverse portfolio becomes similar to equal weighting.

⁷ For the accounting fundamentally weighted portfolios, we instead follow the original universe selection criteria (select the top 1,000 largest stocks by accounting fundamentals) proposed by Arnott, Hsu, and Moore [2005], which is also designed to ensure liquidity. Using the largest 1,000 stock by market cap has similar but less dramatic results.

⁸ The number of stocks by country: Australia—200; Canada—100; France—80; Germany—60; Japan—400; Netherlands—30; United Kingdom—100; United States—1,000; Global—1,000.

⁹ The Fama–French Four Factor Model is an extension of the original Fama–French model, which attributes return to Market Beta, Size (SMB, or small-minus-big), Value (HML, or high-minus-low), and Momentum (UMD, or up-minus-down). This last component was added based on the work of Asness [1994] and Carhart [1997].

¹⁰ Similar to Chow, Hsu, Kalesnik, and Little [2011], we find varying the methods and data frequency for the risk estimates to have no meaningful impact on the results.

¹¹ Minimum Variance was championed in the 1980s by Bob Haugen, during his tenure at UC Irvine. Earlier, in the late 1960s to early 1970s, Haugen and his co-authors empirically documented that portfolios with low volatility stocks outperform the cap-weighted market (see, for example, Haugen and Heins [1975]).

¹² The details of “Maximum Diversification” and of “Risk Efficient Index” strategies can be found in the Choueifaty and Coignard [2008] and in Amenc et al. [2010] papers, respectively. RCEW is based on QS Investors’ Diversity-Based Index methodology. See Chow, Hsu, Kalesnik, and Little [2011] for a review of the portfolio construction strategies associated with the three quantitative strategy indexes described in this section.

¹³ Following Arnott, Hsu, and Moore [2005] the book weighted, five-year average earnings weighted or composite four-metric weighted strategies select top 1,000 stocks using fundamental measures to capture the fundamental economic footprint of the companies’ businesses rather than selecting the top 1,000 based on the market capitalization.

¹⁴ To measure the earnings growth we use five-year average dollar change in reported earnings divided by the average absolute dollar value of earnings over the five-year period. The last fiscal year of the measuring window is two years prior to index construction.

¹⁵ Jeffrey Graham [2012] surprisingly found no alpha for his random portfolio; in fact he found that a randomly generated EW portfolio asymptotically converged on the cap-weighted portfolio in simulation. After reviewing his work, we have concluded it to be a mistake. A more comprehensive study by Clare, Motson and Thomas [2013] of the Cass School of Business, City University London, found alpha for the random portfolio, which is consistent with our result on random portfolios.

¹⁶ In Exhibits 6 and 7 we show selected results for a few individual developed countries. The individual countries demonstrate the same general pattern as we observe in the US or Global Developed markets.

¹⁷ Readers can find a detailed comparison of implementation costs and investability of the popular alternative beta strategies in the paper by Chow, Hsu, Kalesnik, and Little [2011].